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Private Equity Capital Commitments: An Options-Theoretic Risk Management Approach

Andrew Freeman, D. Sykes Wilford

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Private Equity Capital Commitments: An Options-Theoretic Risk Management Approach

Andrew Freeman – Fellow, Centre for Risk Studies, Judge Business School, Cambridge University
D. Sykes Wilford – Hipp Chair Professor of Business and Finance, the Citadel[1]

Abstract

The capital call for a private equity (PE) firm has been described in various modeling approaches almost entirely from the perspective of the investor, from the liquidity implications, in the context of Modigliani-Miller, a Merton type environment, or using forms of Markowitz allocation modeling. In most articles, the nature of the call option (written by the investor and owned by the fund), as it relates to providing liquidity, is assumed. This article narrows the discussion by focusing on the risk of PE firms during a financial or economic crisis. Two sets of options are analyzed. First, we examine the ability to call funds when opportunities arise during periods of market stress. Second, a PE firm's highly flexible ability to "put" holdings to the markets, by waiting for more opportune times to do so, is discussed. Our aim is to better understand the market risk associated with any single PE fund and in aggregate the risk of the PE firm, with respect to these options held by the fund or firm. In an attempt to quantify the risk of the PE fund or firm it is essential to understand from various perspectives the option-like qualities of the contracts that the fund has with its investors. The conclusions should be obvious to the risk manager of a firm or fund, but are often blurred in an attempt to make these investments fit into simple VaR systems or more complex theoretical models. The implicit option-like characteristics create a set of offsets to potential mark-to-market losses as market volatility changes, especially with respect to markets in crisis. The old saw that a financial crisis is a "friend" of the PE firm is vindicated to some extent by the analysis in this article from the perspective of risk measurement and management.

INTRODUCTION

When a private equity (PE) fund is raised, investors give commitments to make an agreed amount of investment capital available at the behest of the fund manager. The contract with the investor allows the PE firm to draw down the commitments via a series of capital calls. In a typical fund with a 10-year life cycle, the expectation is that commitments will have been called fully or almost fully by no later than the end of the fourth or fifth year and that during the second half of the fund's life cycle capital will steadily, if unpredictably, be returned to investors via distributions as a consequence, first, of dividends from investments made, and, second, of eventual exits from past transactions.

This arrangement appears straightforward enough. In fact, applying the lens of options theory suggests that it is anything but. It can shed interesting light on an important strategic question of risk measurement for PE funds, PE firms, for their investors, as well as for regulators and institutions concerned with macroeconomic and financial system stability. Although there are many questions to be addressed vis-à-vis the different regulators, risk managers, investors, and the PE funds themselves, most of these questions link back to the relationship of the PE fund structure to public markets. For example, (1) how vulnerable are PE firms to volatility in public equity markets? (2) In the scenario of an extreme market decline, would PE firms be to any extent immune from systemic collapse? And (3) would that immunity be size-dependent? To answer basic risk-related questions for PE funds or PE firms, we must focus our analysis on the option-like capital call provisions associated with any PE fund structure.

The right to make capital calls is a time-variant call option that is written to the PE fund manager by investors. Deriving its value is complex because the option contains a significant element of liquidity premium; the ability of the PE fund (and therefore the PE firm) to call capital when it is becoming more valuable, in part because liquidity is becoming scarcer, is a powerful right during times of stress. While we can posit with confidence that the value of a PE firm would be negatively impacted by a sharp fall in public market valuations, the call option forms a natural offset both in terms of liquidity and by giving the PE firm (through the funds it manages) the opportunity to buy assets at distressed prices. In "normal" times, the call option has clear value. However, we argue that its value is actually greatest during a crisis and its aftermath.

Further, in contrast to an active investor in public markets with readily tradable securities and consequent exposure to liquidity risk in times of crisis, PE fund managers also have the option to put individual investments in the fund to the market largely, but not wholly, at times of their choosing. In effect, they can to some extent wait out a crisis. Again, this provides an interesting option to the PE fund, and therefore the investor in a fund, relative to a typical equity investment. The combination of options that the PE fund and fund manager hold can potentially be exploited, thereby neutralizing certain risks faced by other types of equity-based funds.

PE FIRMS DURING CRISES: OPTIONS-BASED CRISIS IMMUNITY BUILT IN?

Using a simple schema we can observe that PE funds bring a degree of liquidity to otherwise illiquid corporate assets, and over time they tend to deliver higher return-on-equity (RoE) than diversified portfolios of public equities.[2] For this reason, PE fund managers receive rewards in the form of management fees and carried interest. Their opportunity to deploy liquidity expands across more than one dimension in times of financial or economic crisis when public valuations are depressed. Liquidity by itself becomes more valuable. Assets become cheaper. The range of assets available to a PE fund expands because many more companies will fall into the enterprise value parameters typically written in PE fund governing documents.

We can make the following assertions about the liquidity call:

- PE funds appear to be paid differently for the liquidity they put to work
  - Before a crisis
  - During a crisis
  - Following a crisis
- The value of the capital call option changes during these three periods
  - Before a crisis it can be valued based upon observed prices in the market and low or stable volatilities
  - During a crisis volatility has risen sharply and equity prices are typically lower
  - The demand for liquidity is greater
  - The return on liquidity is greater
- Once a crisis is resolved, PE funds not yet put to work return to a lower volatility-defined value

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2 We will argue that even if the RoE is lower, it may still be preferable due to the risk mitigating nature of the call option provided to the fund and therefore may be a risk diversifier for the typical investment entity.

The capital call option is not like a typical option, which gives the owner the right but not the obligation to exercise and will expire worthless if not exercised. It gives the PE firm the right to call for capital, just like a normal option, but it also includes an element of obligation – the PE firm must invest an amount of the committed capital by a set date or it will breach the governing documents of the relevant fund, triggering investor rights for redemptions, fund closure, etc. However, from the perspective of the PE firm, it has an unusually long time horizon, typically measured in years or months rather than weeks or days.[3] While one can measure the runoff of the options held by the PE firm as a redemption date nears, for illustrative purposes we will consider the options as long dated and not deal with this issue here.[4]

The risk profile of a PE fund is, therefore, correlated with the extent of the value of its capital call options at a given moment in time. As such, the maturity structure of a PE firm's portfolio of funds will ultimately determine the extent of its robustness over the economic cycle, in particular its ability to deploy precious capital during periods of distress. PE firms – managers of the fund – that are unable to make such deployments are at a strategic disadvantage versus their competitors. This insight might help to explain industry dynamics such as fundraising cycles. However, it raises additional non-trivial problems. Some of the large PE firms are themselves listed on stock markets, meaning their own equity is likely correlated with public valuations in ways that may not be fully understood. Moreover, they often have hundreds of funds, some huge some small, across a range of different maturities – some will be newly raised, others winding down.[5]

Further, PE funds in wind-down (i.e., they are returning investment proceeds to investors) also have interesting option-like characteristics, which suggest that PE funds actually own puts, as well as the calls noted above. Because they must sell assets, including via flotations on public markets, they are at first glance vulnerable during times of stress and will suffer from lower valuations and reduced liquidity. In most cases, however, they have considerable choice as to exactly when to do so. In effect, the PE fund owns a set of options to put assets back, either directly to its investors, to other trade buyers or to public markets. However, depending on the precise conditions of a portfolio of funds, some of the put options will retain significant time value because they might not need to be fully exercised for months or even years. In other words, the PE fund might be immune, even if only partially, to short-term crises or even a crisis of considerable duration. In addition, a fund that is winding down will tend to be reducing its leverage, meaning that it is progressively less vulnerable to market conditions. The liquidity call options, combined with the flexibility to put existing assets to the market at the choice of the fund, potentially smooth out risk over time.

Another feature of the capital call option is that by definition its "crisis premium" decays in proportion to the extent that the relevant fund makes investments; more investments means less cash to call. The less capital that remains to be called, the less value the PE (fund) firm can extract from distressed markets. For a typical PE firm that is diversified across time, sectors, geographies, and asset classes the timing option to put an investment to the market certainly mitigates the risk of a particular fund that is winding down, with respect to the declining value of the call option. By maintaining funds at different stages of maturity or drawdown, a PE firm in effect hedges itself against extremes of volatility.[6] This insight offers a new dimension for thinking about the influence of "dry powder" – this is the collective amount of capital available at any one time for deployment by the PE industry. Whereas dry powder is typically, and not incorrectly, viewed as a measure of the competitive pressure on PE firms to do deals at more aggressive prices in good times, it might just as significantly be seen as a gauge of the industry's likely resilience in the event of a crisis.

And, some degree of "crisis" should be viewed as the norm. Karagiannidis and Wilford (2015) suggest that crises in equity markets occur every five to 10 years. However, demonstrably each crisis is different and, therefore, has different implications for valuations, not only with regard to the underlying assets in a PE fund, but also to the call option, owing to typically wide swings in volatility. For example, prior to the recent financial crisis, the volatility of the S&P jumped from about 16% (average for the previous six years) to almost 60%, before returning to the lower post-crisis levels.[7] A glance at the historical movements in the VIX index shows a similar picture. In the mid-2000s it was in the low teens before spiking to above 60 during the crisis. And even after the crisis, spikes occurred that reached the high 30s.[8]

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3   For our purposes we will not attempt to model the implied run-offs in the value of the call due to the contractual obligation embedded in the PE Fund governing documents that committed funds must be invested by a set date. Any attempt to measure the changing values of the call option must a priori take this obligation into account. Indeed, in valuing the call option, one should consider the extent of decay as a fund reaches the point where it needs to be nearly or fully invested.

4   Our analysis is necessarily limited. We do not examine the embedded options referred to above. Some PE firms begin fund-raising on a next-generation fund once the existing fund passes its investment thresholds, since the ability to refresh its contingent capital via closing of a new fund appears to be critical to the PE firm's ongoing financial resilience.

5   We do not address the potential for a mispricing of the publically traded funds in this analysis, but it may be the case that the market trades these firms at a much higher risk premium, especially during a crisis, than is warranted. Further research is needed on this interesting question.

6   A much more formal approach to the issues related to valuations relative to publicly traded positions is found in Sorenson et al. (2013).

7   See Wilford (2014) for more discussion of the relevant volatility measures.

8   See http://www.cboe.com/delayedquote/advchart.aspx?ticker=VIX.

Spikes in volatility are the "lifeblood" of options. It is these spikes that make the call/put provisions of a PE firm so valuable; in essence providing a cushion to the risk associated with funds and, by implication, PE firms. Interestingly, size matters with respect to how the risk is mitigated by options ownership for the PE firm. Indeed, small PE firm (and perhaps venture) assets are more likely to become attractive to large PE firms during a crisis because they may carry less options-based immunity and will, therefore, decline in value to a greater extent than large PE firm assets. This helps to explain why the valuation challenge is so severe.

Whereas a generalized model is not attempted in this analysis, careful illustration of the options implicit in the structure of the PE fund can demonstrate the potential this perspective offers for risk mitigation.[9] Using a simple construct, the PE firm's risk may be mitigated by the aggregation of the embedded options in the funds that the firm manages.[10]

VALUING THE OWNERSHIP OF THE PE CALL OPTION – FROM WRITER TO THE FUND, AND THE PE FIRM

It should already be clear that valuing the PE call at the fund level is required before statements about the value of these options to the PE firm can be made. Any application to risk measurement must take into account the different maturity characteristics of individual funds in such a way that these can be aggregated into a meaningful picture of a PE firm's portfolio where the firm operates multiple funds.

Moreover, the term "PE firm" is something of a misnomer, because the leading firms have increasingly diversified across asset classes since the recent financial crisis. As banks reacted to new regulatory capital and liquidity requirements, they reduced their lending to parts of the capital markets, including middle-market commercial lending and mezzanine financing. This created an opportunity for PE firms to offer funding in non-pure-equity areas of companies' capital structures, including the flexibility to create combined equity/debt solutions. The addition of debt capital instruments created a new layer of complexity to PE risk profiles and has yet to be clearly analyzed. But, to the extent that a PE firm has non-PE assets, then it will react differently to external market conditions, particularly during periods of stress or crises. Existing debt structures, especially mezzanine financing, imply unique pricing problems, almost all of which are based on options theory. As per above, the risk of government debt rose dramatically during the crisis as measured by volatility.[11] Based upon Merton (1974), corporate debt can be viewed as a put option on the value of the firm plus a close to

risk-free government bond. Combine this fact with the right to invest in such debt at moments of extreme risk (when the put is most negatively affected by a sharp rise in volatility) and the call provision again offers a cushion to the risk of a PE fund; potentially offsetting the negative implications for increasing spreads on high yield instruments that may be held in, say, a mezzanine fund.[12]

To simplify matters we note that owning the right to call – whether it is to invest in equity directly, indirectly through mezzanine debt, leveraged through warrants that typically are attached to mezzanine transactions, or any combination thereof – is valuable and may offset some of the negative implications for a fund or the fund manager with respect to the inherent risks resulting from a crisis movement in market prices (and their declines). Particular circumstances will dictate the methodological approaches necessary to measure the value of these options correctly; the key is to recognize that they are valuable and that their value changes radically during a crisis in ways that may be inversely related to movements in public markets.

Will the investor deliver on the call and PE firm (fund) implications?

An underlying assumption of an options-based approach to measuring the volatility of PE firm valuations is that the capital call options change in value over time depending on market conditions and related asset prices, but that they can always be exercised because investors are contractually obliged to honor all capital calls. Investors subscribe to funds governed by by-laws and agreements that are set out in advance. In effect, they would be in serious breach of contract were they to refuse to meet a capital call. The same would apply if they were unable to meet a call because of distress and a subsequent lack of necessary liquidity. Under the terms of a typical fund, the PE firm would then have the right to eject the investor from the fund and seek a secondary sale of the relevant commitment. It could also revert to legal proceedings for breach of contractual obligations, particularly if the investor's unwillingness to pay could be shown to have damaged the interests of other investors in the fund.

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9 It can be argued that a "building block" approach to measuring the risk mitigation implicit in the call and put options can yield very practical results in pricing and measuring risk. This is based on the building block focus of Smithson (1998) in valuing derivatives and financial structures.

10 For a typical PE fund investor knowledge about the degree of risk mitigation at the firm level (perhaps size is correlated, ceteris paribus) may be important information in choosing a fund in which to invest.

11 We are ignoring direction of prices and, therefore, correlations to the S&P.

12 See Jensen and Meckling (1976) and Wibaut and Wilford (2009) for theoretical analyses, as well as implications of volatility spikes on corporate debt.

In practice, however, would a PE firm resort to legal action? It would have to take into account overlapping factors. Suing an investor in distress might lead to serious reputational harm for the PE firm, even if the legal case for doing so were ironclad. The investor concerned might have investments across multiple funds, meaning it would have an important relationship with the PE firm that would then be in jeopardy. A lawsuit could also send a negative signal to other investors, both existing and prospective, decreasing their appetite for future commitments. These real costs might outweigh the cost of the missing capital that was called and not delivered. Assuming the capital call was to fund an investment, the PE firm might be better off making good the opportunity from its own capital in order to drive home its advantage.

Depending on the circumstances, then, a PE firm might decline to exercise its right to enforce a call option. Again, this further complicates the underlying problem. At some point, the risk can become so high that the writer of the call may default on their obligation. To the extent that the PE firm's utilization of this option to measure (offset) the increased risk as suggested by the market is compromised, this benefit is lost. From an options-theoretic perspective, such an event suggests that a risk manager should consider vega risk for the PE firm, although the lack of information about the circumstances under which default on the contract may occur means that it is difficult to measure such a risk. In practice, the recent financial crisis witnessed many institutional investors in PE having to find strategies to meet their commitments that on the surface caused significant difficulties in valuing the assets of the institutions and perhaps led to some wealth destruction.[13]

A SIMPLE ILLUSTRATION OF THE VALUE OF THE OPTION TO THE PE FIRM

Previously, we have stressed the actual complexity of the liquidity call embedded in the PE fund contracts. In effect, to apply this methodology to an existing fund, in an attempt to enhance any risk measurement system, each of these complex options must be broken into its component parts and recombined into whatever system of risk measurement is appropriate for the PE fund or firm. Alternatively, it would be possible to create an overall model that considers all of the risks inherent in the contract.

Our analysis will focus on some of the simpler aspects of the call provisions (as well as the put options implicit in a fund with respect to timing of placement of invested capital). In doing so, we begin the process of measuring the implications of the ownership of the call (put) options by the PE fund and in addition, the firm. Armed with simple snapshots of the change in value of the option to the PE fund we can then suggest some logical implications for risk measurement and perhaps the management of risk.

The liquidity call – a simple approach
The impact of a financial crisis on the implied value of the call can easily be illustrated with a simple pricing model. Using measures of volatility from Wilford (2014) as guides, we can calculate the value of the call from the perspective of the fund by simply allowing volatility to vary with the pre, post and crisis periods. This can provide us with snapshot comparisons of the value of a call.

	Pre crisis	Crisis	Post crisis	Recent period
Volatility	16%	57%	17%	13%
Value of call as a percentage of the notional principal	18.54	49.91	19.44	16.25

Table 1 – Call option value

As a first exercise, let us assume a five-year call period (typical of many funds) and set the strike price at 100% of the exercise price. Using an interest rate of 2% a rough calculation of the value of the call can be estimated. For our purposes, we simply desire to understand the implication of the rise in volatility on the value of the call.

This illustrates how a crisis will drive up the value of the call. In this exercise, each period was calculated assuming the call starts during the relevant period. This is not reality, of course, but a near tripling of the value of the call illustrates the point that the holder of that call has had an increase in value just when it is needed most

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13 See, for example, Phalippou and Westerfield, 2014, "Cash-poor LPs face capital-call pressure," citing Private Equity Insider, footnote 3. November 5, 2008: "Brown University, Calpers and Carnegie Corp. are suddenly finding it hard to meet capital calls from private equity fund managers (…) A growing set of limited partners find themselves short on cash amid the financial crisis and thus are scrambling for ways to make good on undrawn obligations to private equity vehicles. Among those in the same boat: Duke University Management, Stanford Management, University of Chicago and University of Virginia… Brown, whose $2.3 billion endowment has a 15% allocation for private equity products, is apparently thinking about redeeming capital from hedge funds to raise the money it needs to meet upcoming capital calls from private equity firms. That's similar to a strategy that University of Virginia is employing… Carnegie, a $3.1 billion charitable foundation, is also in a squeeze. Its managers have been calling on commitments faster than expected, while distributions from older funds have slowed down, creating a cash shortfall. As for Duke, the university's endowment has been named as one of the players most likely to default on private equity fund commitments. That partly explains a massive secondary-market offering that the school floated last month, as it sought to raise much-needed cash and get off the hook for undrawn obligations by unloading most of its $2 billion of holdings in the sector… Some of the bigger investors are considering tapping credit facilities to meet near-term capital calls."

from the perspective of measuring and managing risk.

Alternatively, once a crisis passes and the rest of the investment portfolio of a PE fund or firm regains some of its mark-to-market value, the value of the call falls.[14] The opposite movement in risk measures will never be symmetrical in nature, but the point should be clear. Just when a fund may find itself under pressure from falling value of existing holdings, the opposite is occurring with respect to the call provisions and perhaps sufficiently to have a significant impact on overall values of the fund (or firm).

To make the exercise more meaningful, let us now assume that a firm entered into a contract with an investor one year before the onslaught of the financial crisis. After one year the call will have had time decay. Thus, instead of the call being valued at nearly 50% of the notional principal, as per above it, will have a value of 5% less. Of course, as the time period changes, so will the value and the decay factor will eventually have a waterfall effect on the gains from an increase in volatility. Still, the fundamental point holds that during a crisis the value of the call may offset to a great extent the implied mark-to-market decline in the value of the PE fund's holdings resulting from the onset of a crisis.[15]

Implied price	100	80	60	50
Value of call	50%	52%	59%	64%

Table 2 – Pricing the strike

As noted above, a crisis might offer better opportunities for a PE firm than a bull market. If this is the case, then the exercise price of the call and the implied spot price will vary with a shock (implied price means the percentage of the underlying price). Indeed, one could argue that as the general market decline occurs so will the value of the company to be targeted. One way to model this then is to allow the exercise price to decline in the option calculation. During the financial crisis, equity markets plummeted by some 50%, more or less, depending on the market. In a simple risk analytics mark-to-market model one could argue that the existing investments of some PE firms would also be impacted by 50%. If so, then how is the crisis the lifeblood of PE investing? The answer is obvious; with the effective cash available due to the call provision the PE fund can buy assets at these reduced prices. Put another way, from a risk management perspective, the call offsets, to some extent, the decline in mark-to-market prices.

Let us now assume a 50% fall in prices and examine what happens to the option as it is effectively coming more into the money. In our case above of a one-year time decay, the value of the option moves from approximately 45% to 64%, or an increase of 50% in value.

Such a large fall in general market prices is unusual, so perhaps by doing the same exercise but holding pre and crisis volatilities the same while allowing the implicit fall in the exercise price we can see more clearly the risk-mitigating potential of the call for the PE fund (firm).

The implied price decline of a targeted investment for a fund that would require a call may or may not fall so significantly with the rise in volatility.[16] It is, however, a factor that needs to be considered in understanding the overall impact of the call provision on risk measurement and obviously there are implications for risk management of the PE fund or firm.[17] In a simplified form, at a market price of 50 the fund would be able to buy twice as much as if the price was 100, but from an options value perspective the fact that the option calculation implies that it is much more in the money suggests that its impact is less relative to the underlying impact of lower prices.

Clearly a fund that has just finished fund raising and is about to begin deploying capital has a huge advantage over one that is nearing maturity with respect to the value of the call for risk measurement and management purposes. One may think of the combination of the call versus the percentage of the call contracts called as a vega risk problem. For the sake of simplicity, however, we think that the measurement of the risk of the actual investments resulting from a crisis should be considered separately from that of the call itself. The overall risk of the fund can thus be aggregated based on the percentage called (and therefore invested). Although in combination it may be intellectually interesting to look at the percentage of the call outstanding versus what has been called as a vega calculation problem, the simpler aggregation of implied values should yield similar and intuitively understood results, especially when combined with mark-to-market or other valuation implications from general market movements in underlying prices.[18] Again, the problem is made more complex by the fact that PE valuations are rarely derived solely from observable market prices, but are constructed

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14 Implicitly we have assumed that a serious crisis would be correlated with a decline in publically traded equity prices and a recovery would imply a movement toward the long-term trend of rising equity prices.

15 In any attempt to model the timing of a call occurring, one would need a simulation approach based on some random process. Here we are suggesting the basic concept, not any particular time when a call would be made.

16 Logically, these estimates of the impact on the value of the call due to a drop in the price of the underlying are subjective.

17 Fairly sophisticated modeling by Sorenson et al. (2013) integrates some of these issues. However, from an applied perspective we believe that it is necessary to consider the subjective nature of the risk measures for a crisis. Looking for perfect or near-perfect quantitative measures will often miss the point in practice.

18 Whatever system is chosen for measuring the variable risk of an asset, it must be considered along with the value of the options held by the fund because it has an inevitable implication for the correct input values for the option pricing scheme utilized.

by a process of peer comparisons and earnings discounting techniques.

Valuing the implied put option

So far we have focused on the value of the call to the PE fund (firm); however, as noted, PE firms have another set of options: they can time the putting of the private investment to the market. Tables 1 and 2 provide an insight into the value of this timing as well. It is precisely when the market is most risky in a crisis, with an overall decline in opportunity to sell the existing assets, that owning the put is most valuable to the fund (firm). A simple illustration of this is presented in Table 3.

Again, the value occurs in the crisis period when the market has been hurt so badly and the fund manager may be forced to write down the value of the holdings not yet put to the market. As the market recovers, the value of the put declines, as one would expect. For the risk manager of the PE fund or firm both of these implied options – call and put – are helping to control risk just when normal mark-to-market would suggest a large write down in the fund's value. The natural cushion imbedded in the PE structure itself mitigates the overall market risk associated with the investments.

It is interesting that from a risk management perspective both the puts and the calls implicit in the structure of PE funds mitigate the overall risk of a crisis significantly while not hurting the long-term performance of the fund. Existence of these options could easily offset declines in the market itself, and depending on the amount of the fund invested, but not brought to market, the crisis could actually raise the value of the PE fund while the market is in disarray. As such, the notion that crises are the lifeblood of the PE firm may not be a strange comment at all, but is in fact supported by the large movements in value of the options implied in the structure of the funds. Of course, the value of the call, as well as the put, will depend upon the percentage called and invested at any point in time, but this is mechanical and can easily be considered by the PE fund risk manager as well as aggregated at the PE firm level.

The correct value of the put, however, is much more difficult to

determine. In Table 3 we made simple assumptions about the strike price for the put. In simulating over any meaningful period of time, the problem becomes much more difficult. What is the strike price to be considered? Is it the market price, some forward price, etc.? Obviously, if the market simply goes down and the investment itself cannot be taken to market, then eventually the value of the put will go to zero, no matter the volatility. Alternatively, the strike price could be held at its accounting value – what was paid for it or 100 – but this seems obviously incorrect in the case of a drawn out market downdraft. Equally, in the case of a significant rise in the market, choosing a strike price of 100 would imply that the put becomes worthless – we would never simply put it to the market at 100, so in principle the strike should rise along with the market price. Again, this assumption has obvious flaws in the event that we attempt a simulation of the put option over time. Our analysis is deliberately simpler in order to illustrate the value of the put under different conditions, so "knowing" the correct strike price is less of an issue.

GIVEN THE RISK, ARE INVESTORS PAID SUFFICIENTLY FOR THE OPTIONS THEY WRITE THE PE FIRM?

By definition, investors consent to writing a call option to the PE firm. According to options theory in a conventional traded market, any increase in value of the option to the PE firm must be offset by the same loss of value to the investor. During a crisis when volatility spikes, the option writer should feel that there is an implied drop in the value of the portfolio from which it was written, i.e., they have incurred a loss in favor of the PE firm.

Anecdotal evidence suggests that this is not the case. As one foundation manager has queried, "doesn't PE's lack of trading help dampen our risk?" There are several reasons why this could be the case. One relates to the obvious fact that PE firms largely buy non-traded equities, so the options noted above are intuitively considered. They may understand that when there has been a large general loss in public markets there are significant opportunities for PE firms to lock-in long-term bargains, so they tolerate the call option as part of the PE offering of outperformance, including its special role in times of crisis. Finally, the opportunity cost of the option they have written may be offset in some cases by the implied put's rise in value (per Table 3), at least to some extent, for the funds already called and invested. For the moment we ignore this point, although it is critical to the statement above, and we focus mainly on the call written.

If any of these arguments are correct, then the writer of the call option (investor) does not have a decrease in value when volatility rises to the extent that would be the case for a normal call. Purely

	Pre crisis	Crisis	Post crisis	Recent period
Volatility	16%	57%	17%	13%
Value of put as a percentage of the notional principal	9.25	40.52	10.05	6.86

Table 3 – Put option value

quantitative assessment of the gain/loss trade-off would struggle to reflect this complexity. We believe the implications are case specific. To find the key to why an otherwise asymmetrical deal in favor of the PE firm is not generally contested by investors, we consider the general conditions of the portfolio of the investor. The written call provision will have to be funded but how will it be funded may vary. In a simple valuation model, one would assume certainty in funding from the signing of a contract. In reality, actual investment calls come unevenly through the investment period of the fund. As we have shown above, the value of the call to the PE firm can be calculated given certain assumptions and information, but whether there is negative value to the investor, and, if so, its extent, is not nearly as clear.

Let us assume three possible conditions for a PE investor committed to a fund. At this stage we make no assumption as to the lifecycle position of the fund, but we can observe that it will still be in its investment period so that the call option has not expired and therefore retains value:

■ The investor holds cash or T-bills "in case" of a call
■ The investor holds investments with a beta of 1 that will have to be liquidated when a call is made
■ The investor uses a readily available line of credit to fund the call – behind the PE firm's contingent capital lies another supplier of contingent funding in the form of a revolving credit facility

Let us now examine each condition in turn.

Condition 1

Here, the investor obtains the expected return on cash or cash alternatives, so has, in effect, a zero risk-return position (although as we know from recent history this might not be the case in reality, as the concept of a "risk-free asset" has been shown to be of theoretic, but not practical use). The call option does have an opportunity cost, but to the extent that the Treasury position reflects the intended asset allocation there is no opportunity cost, or one that is limited. In other words, large investors with small allocations to cash might routinely hold sufficient liquidity to be able to fund calls even in times of distress. The risk of not being able to meet a cash call is much greater for smaller investors. Similarly, pension fund investors with predictable cash flows are relatively well placed to predict their cash position and to have contingencies for capital calls. They hold a quasi-permanent call option over the contributions from fund members by way of monthly payroll, so even in crises their ability to raise cash is stable. In the real world, we would need to parse the investor base to determine which PE firm might have a particular vulnerability to its investors' collective cash position.

In our stylized simple example, if volatility rises, creating a higher value to the PE fund for the call option, then the investor will not see a consequent reduction in the value of its portfolio.

In Condition 1, the change in value is not negative for the investor. As the call rises in value for the PE firm, the investor does not necessarily lose value. An increase in volatility, as during a period of crisis, does not cost the investor, but it does create a positive value change for the PE firm that should ultimately benefit the investor (assuming the PE firm can take advantage of the crisis). Investment funds that meet Condition 1 logically have gains to trade by seeking investments in PE funds.

Condition 2

The investor maintains an investment in a beta of 1 investment. Here, we choose a beta of 1 for simplicity, but the logic carries for other equity-type choices, hedge funds and other alternative assets, provided the betas are known.

If volatility rises in the market due to a crisis, equity prices will tend to fall sharply. In this case, the value of the call to the PE firm rises, but the value of the assets the investor has to sell in order to fund the call falls accordingly. This is a fine example of a risk or liquidity cascade, where a supposedly safe asset must be sold in distressed circumstances, leading to further falls in asset values and a further reduction in liquidity and so on. Conventional risk management traditionally underestimates the impact of such cascades.

In extremis, this condition could mean that the increase in the value of the call option to the PE firm is reduced or negated by the impact on the ability of the investor to liquidate other assets in order to fund the call. The critical moment is when the investor proves unable to fund the call because of the compromising of its 1-Beta assets. Some investors will be able to fund the call using emergency capital or other measures (unanticipated sale of other assets/emergency borrowing against collateral – see below Condition 3). Let us assume for the moment, however, that it is funded, so as not to introduce a new variable in considering the option value.

The investor's loss (realized in forced liquidation during the cascading market) is thus the PE fund's gain. To the extent, however, that the crisis enables the PE fund to invest at better prices, thereby providing a better return to the investor due to the provision of liquidity during a liquidity-challenged period, then there may be some positive offset to the investor. In the end, the impact could be felt more through the carried interest agreement than through the declared returns on the fund. Even in this case, assuming funding occurs there may be gains to trade for the investor; the extent of the gains due to a risk offset is not easily discerned.

Condition 3

If the investor funds the call via borrowing with established lines of credit, then the offsetting value may be different depending on:

- The cost of creating the line of credit at the time of writing the call to the PE firm
- The interest paid on the credit line once the funds are called
- The possible alternative uses for the credit

Although in theory the capital-structure argument should not change the value of the negative call to the investor, it could do so depending upon the three issues above.

If the investor obtains a guarantee over the necessary credit lines during a period of low volatility, then an increase in the volatility of the market may mean that the call option it has purchased rises in value simultaneously, although not necessarily symmetrically, with the call it has written to the PE firm. Hence, a significant change in volatility may not imply a loss to the investor equivalent to the size of the gain to the PE firm.

If the investor has to borrow during a crisis because it had not secured its line of credit at the time of commitment, then the situation is entirely different. Now the investor must incur the cost of additional volatility in its purchase of a now more expensive credit line and this will at least partially, and possibly fully, offset any advantage it might derive from the gain in value of the PE firm's call option. This assumes that the PE firm is indeed able to find more attractive deals in which to deploy the contingent capital during distressed markets. If it does not, then the net result for the investor in this condition is even worse.

If the investor can deploy its available credit more efficiently, then there is an opportunity cost resulting from having written the call to the PE firm, assuming the capital call takes precedence over other opportunities for purchasing distressed assets and that the call will be met. Does the increase in the value of the call for the PE firm preclude the investor from using its credit to purchase Beta under duress? Is the investor in fact able to make the decision to divert otherwise committed funds to the PE firm towards other distressed assets? We can see that this will be highly idiosyncratic. A few large investors might have the flexibility to go beyond the capital call and make distressed asset purchases of their own – this includes taking advantage of the opportunity to "double down" on public equity markets. Many, however, will lack the optionality to do so.

Perhaps the true value of the capital call option is that it passes from the investor to the PE fund (firm) the opportunity to purchase distressed assets during crises and thereby tends to enhance fund returns, such that management fees and carried interest are "justified." This is because the eventual overall outperformance across a 10-year cycle, riding through any crisis or crises, is notable.

Summary of investor conditions

In Condition 1, it appears that the gain to the PE firm does not have a proportional offset loss to the investor and may not imply any loss in value to the investor if the allocation to cash or Treasuries is not affected by the investment decision.

Under Condition 2, the investor is, in most cases, losing, because the PE firm is gaining value with a crisis or a sharp increase in volatility (this was clearly the case during the recent 2007-09 crisis, for example). This may be mitigated to some extent by the richness of the opportunities created for the PE firm. From a risk perspective, however, the investor has written an insurance policy to the PE firm and bears the risk.

Under Condition 3, multiple outcomes are possible with respect to the size of the loss for the investor given the gain for the PE firm due to a rise in volatility. This is because the "insurance" against having written a call option to the PE firm contemporaneously takes the form of buying a call from another provider of credit. If the option to obtain credit is purchased in a low volatility environment then it is clear that such an arrangement will have value, which may offset the gain in value to the PE fund due to a rise in market volatility. If, however, secure funding to cover the call is obtained during the crisis, gains from trade may be lost due to increased funding costs directly incurred by the investor.

For each of these cases, however, over time the call's rise in value in a crisis will be offset to the extent that part of the funds have been called and replaced by the put that they now own, supporting the notion that the overall implied risk in the PE fund itself should not be managed in the same manner as general market risk. Although the embedded options in the PE fund process are complex, our simple analysis suggests that they may offset much of the downside risk normally associated with investing in equities. If this is the case then, depending on the conditions above, PE fund investing may be less risky to some extent than one would conventionally think. Even if the long-term returns are simply those of the market, then the PE fund would have "correctly measured" a much better information or Sharpe ratio than market investing, arguing for significant inclusion of such investments in diversified portfolios. And to the extent that the option characteristics are orthogonal (or low to negative correlation with general equity prices) such investments may decrease significantly portfolio risk at the margin.

SUMMARY REMARKS

We have assessed the complex optionality of the contract between a PE firm and its investors during the period of fund commitment and subsequent investments. The value of the capital call must be viewed as an asset for the PE (fund) firm. How the PE (fund) firm itself will change in value will depend upon the degree to which the call option has decayed with time or, more interestingly, with the possibility of a non-funding of part of the commitment, thereby creating the potential for vega risk associated with the specific PE fund. Across multiple funds within a single PE firm this is a non-trivial problem in the management of risk for the PE firm itself.

The positive value of the call option in a traded market would, in theory, be of equal portion negative for the writer of the option under normal circumstances if the markets to trade them were fully developed. We have shown that in the PE world it seems clear that risks and values are asymmetrical. For the investor writing the option, the trade-off appears to depend on how the call is funded. It seems clear that if the call is funded by the investor merely as a way of increasing its exposure to equity markets (for example, maintaining a beta exposure while simultaneously writing the call), then an increase in risk (volatility) will hurt the writer via losses, potentially severe losses, on the beta portion of its investment during a crisis. The increased volatility should be reflected in the overall increase in the risk of the investor's portfolio, suggesting an opportunity for more efficiency in the market to develop. If this is indeed the case then a greater development of the PE fund market would help complete the proper pricing of these embedded options vis-a-vis the

portfolio allocation to PE funds by investors relative to typical asset allocations suggested by naïve models.[19]

Under the other funding choices for investors, risk levels are not so easily understood. If the call is funded by cash equivalents or highly liquid government bonds, then an increase in market volatility may benefit the investor, at least with respect to this marginal risk portion of the portfolio. Whereas the value of the call option to the PE firm rises and therefore its risk falls, the risk savings to the PE firm do not come at the expense of the investor. Credit funding may or may not have a similar effect, depending on the planned capital structure of the investor's portfolio and leverage. It is all about the extent to which the relevant parties have thought ahead and created risk-tolerant positions.

From the perspective of the PE firm holding the capital call option, ceteris paribus, increased volatility (or a crisis) implies an increase in the value of this key asset of the fund and thus the firm. Gamma risk is positive. Only if there is a cap on the ability to call funds from investors should one consider the vega risk associated with the option. This is a disaster risk a PE fund may consider and treat any vega risk accordingly.

From the perspective of this narrow measure of risk for the PE firm itself, owning the call may simply be an offset to rising risk in times of crisis or high-volatility markets. And once the put provisions of the contracts are better assessed it becomes less and less clear when the investor is hurt by a crisis. No doubt the Condition 2 investor has more at risk; perhaps this was the real world point faced by the investors noted above in footnote 7. In other cases, however, just as the existence of the options smooth out the implied risk for the PE fund (firm) during a crisis, it may also be the case for the investor under certain conditions. Table 4 lays out the crisis situation and the implications for the crisis at various points in the life of a fund with respect to the imbedded options in the fund.

CONCLUSIONS

Recognition by investors and managers of the implicit options in private-equity investing is essential to determining the risk associated with those investments, particularly as it relates to general market risk. A simple approach to breaking out the options implicit in the contracts entered into by PE fund (firm) managers and investors

	Zero % called	Half called	Fully called
PE fund firm call	Highly valuable	Valuable	No value
PE fund put	Zero value	Valuable	Highly valuable
Investor call condition 1	Neutral	Neutral	No value
Investor call condition 2	Negative in value	Negative in value	No value
Investor call condition 3	Neutral to negative in value	Neutral to negative in value	No value
Investor put condition 1	Potentially valuable	Valuable	Highly valuable
Investor put condition 2	Potentially valuable	Valuable	Highly valuable
Investor put condition 3	Potentially valuable	Valuable	Highly valuable

Table 4 – Assume a crisis

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19 This observation was highlighted thanks to Charles Smithson who pointed out that swaptions "completed" the corporate callable bond market where calls were typically underpriced before the development of the options market.

yields interesting insights into each group's risk profiles. It would be a significant mistake of risk analysis to ignore the implicit options embedded in the PE fund. For the risk manager or any interested party (such as an investor into publicly listed PE firms) recognition of the role these options have in mitigating the market risk of the fund is crucial.

Although we have taken a simplistic approach to analyzing the options themselves, this does not negate the implications that a VaR, or any similar market-based, approach to risk valuation of a PE fund (firm) is missing many of the most important factors. In spite of the widely held beliefs that many "smart" investors were damaged by their reliance on PE fund investing during the crisis, this need not at all be the case for most investors. We can see why Condition 2-type investors would be damaged by being over leveraged. For other investors, it is not clear, or in some cases quite the opposite; the crisis makes these investment vehicles risk-reducing on the margin when considered in a portfolio context.

Moreover, given the conclusion that the existence of these options naturally mitigates risk, more efficient investment portfolios may be created by including PE fund allocations. The "correct" proportion of an allocation is dependent on many factors (some noted above). However, if PE fund risk is much lower than is priced by the market in part because there is poor understanding of the options involved, then systematic under-allocation to PE funds (and under-pricing of PE firms' equity) is likely.

Although we have used snapshots of period volatility and assumed time periods to measure the value of both the calls and later the puts owned by the PE fund and ultimately the PE firm, these snapshots highlight the need for an options-theoretic approach to measuring risk for the PE fund (firm). A next step would be to use a typical time horizon for a fund, a typical period to call the funds, a typical period to place the funds and a typical period for returning investments and earned returns to the client, as well as reinvestment along the way. This requires considerably more sophisticated analysis, but is potentially feasible. One could create a rolling valuation of the implied options, taking time snapshots over the life of the fund. Using a simplistic backward looking volatility pattern would be a first step to seeing how the values of the options unfold over time.

Further, crisis modeling can be utilized. We chose the recent financial crisis, but others, such as the LTCM crisis, or Asian Contagion, or the 9/11 shock, could also be employed. In all cases, the ownership of options by the PE firm may significantly mitigate the shock and the downward implications of such tail events. Any risk management system for the PE fund or firm should include the consideration of fully pricing these options in the event of a shock. This has

implications for PE firm risk managers who must consider stress testing as part of their regulatory requirements.

One must also note that the typical PE firm has a portfolio of funds. As such, the long-dated options it holds are also a portfolio. A portfolio of options will likely be less valuable than simply adding up the value of the individual options themselves, an effect that is another avenue for future research.

Finally, we want to stress the difference in risk measurement and risk management. Implementation of a strategy to capture the value of the options that are owned by the PE firm goes beyond knowing that they have an offsetting risk capability during a crisis. The use of OTC or exchange-traded baskets of options may provide a way to capture this value during the heat of a crisis. Depth of markets, types of baskets that may be created, and the ability to manage effectively the vega and theta risk of the options sold to capture the value embedded in a volatility spike are issues that need addressing if risk measurement is to be turned into truly effective risk management.

We expect that further study will uncover academically sound analytics to support the simple notion that a crisis is indeed the "best friend" of the PE fund.

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